Effects of Virtual Manipulatives on Student Achievement and Mathematics Learning

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ABSTRACT

This paper is a meta-analysis that synthesizes the findings from 66 research reports examining the effects of virtual manipulatives on student achievement. Of the 66 reports, 32 contained data yielding 82 effect size scores with effects of virtual manipulatives on student achievement. The 66 reports also contributed to a conceptual analysis of affordances that promote mathematical learning. The results of the averaged effect size scores yielded a moderate effect for virtual manipulatives compared with other instructional treatments. There were additional large, moderate, and small effects when virtual manipulatives were compared with physical manipulatives and textbook instruction, and when the effects were examined by mathematical domains, grade levels, and study duration. The results of the conceptual analysis revealed empirical evidence that five specific interrelated affordances of virtual manipulatives (VMs) promoted mathematical learning. These five specific researcher-reported affordances included focused constraint (i.e., VMs focus and constrain student attention on mathematical objects and processes), creative variation (i.e., VMs encourage creativity and increase the variety of students' solutions), simultaneous linking (i.e., VMs simultaneously link representations with each other and with students' actions), efficient precision (i.e., VMs contain precise representations allowing accurate and efficient use), and motivation (i.e., VMs motivate students to persist at mathematical tasks)

Moyer, Bolyard and Spikell (2002) define a *virtual manipulative* as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). Many virtual manipulatives (or dynamic objects) commonly used for mathematics today are movable pictorial representations in the form of applets (in Java) or apps (for the IPad). In addition to virtual manipulatives, which are based on physical objects, some computational media have created new representational forms (e.g., dynamic geometry programs, graphing applets) (Kaput & Roschelle, 1998). Virtual manipulatives, described as dynamic objects, should not be confused with the broader category of *digital learning objects*. Digital learning objects are defined by Wiley (2000) as "any

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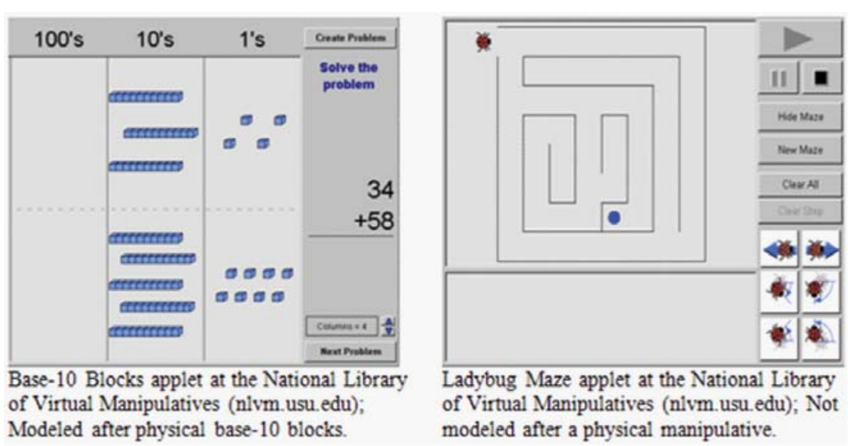
digital resource that can be reused to support learning… Additionally, learning objects are generally understood to be digital entities deliverable over the Internet, meaning that any number of people can access and use them simultaneously" (Wiley, 2000, p.7). Digital learning objects cover a much broader scope of media than virtual manipulatives.

A variety of websites feature collections of virtual manipulatives, including the National Library of Virtual Manipulatives (NLVM) (http://nlvm.usu.edu), National Council of Teachers of Mathematics (NCTM) Illuminations (http://illuminations.nctm.org), and Shodor Curriculum Materials (http://shodor.com/curriculum/). The libraries contain interactive Java applets designed to focus on a single mathematical concept. Some of the virtual manipulatives in the libraries are based on common physical manipulative objects (e.g., base-10 blocks or platonic solids at nlvm.usu.edu), while others have no physical counterparts (e.g., turtle rectangles or scatterplot at nlvm.usu.edu). See Figure 1.

The National Council of Teachers of Mathematics (2000) reports that "work with virtual manipulatives can allow young children to extend physical experience and to develop an initial understanding of sophisticated ideas like the use of algorithms" (NCTM, 2000, p. 26-27). Sarama and Clements (2009) report that "computer manipulatives provide unique affordances for the development of integrated-concrete knowledge" (p. 147). Sarama and Clements (2009) note, "What gives integrated-concrete thinking its strength is the combination of separate ideas in an interconnected structure of knowledge. For students with this type of interconnected knowledge, knowledge of physical objects, actions performed on them, and symbolic representations are all inter-related in a strong mental structure" (p. 146). Virtual manipulatives were designed to connect pictorial representations, actions performed on them, and symbolic representations to highlight mathematical concepts and focus the attention of the learner on the mathematics to be learned. For over two decades, researchers have documented the effects of virtual manipulatives in mathematics instructional treatments. Yet, to date, there has been no attempt to synthesize this research base. The purpose of this study was to directly address this need. The findings from 66 research studies were synthesized in a meta-analysis to examine the effects of virtual manipulatives on student achievement and learning.

Figure 1. In these examples from the National Library of Virtual Manipulatives, the applet on the left is based on a common physical manipulative (i.e., base-10 blocks) and the applet on the right has no physical counterpart



PRIOR THEORETICAL WORK ON COMPUTER-BASED TOOLS FOR MATHEMATICS LEARNING

There is a long history of using computer-based tools for mathematics learning. In the 1980s, Thompson (1985) defined a microworld as "an ideal environment composed of objects, relationships among objects, and operations that transform objects and relationships" (p. 465) and a computer microworld as "a 'playground' somewhere between concrete models and abstract formalisms for developing intuitions of abstract concepts" (p. 466). Embedded in Thompson's early ideas about computer-based tools for learning mathematics are foundational ideas that underpin many of the computer tools in use today for mathematics learning: relationships among objects, operations that transform objects, and connections between models and abstraction.

Relationships Among Objects

The virtual manipulatives in use today for school mathematics contain representations of objects; learners can perform operations that transform those representations of objects; and the representations of objects are linked with symbolic notations of abstract concepts. Many years ago, Kaput (1986) suggested that the linking feature in many forms of computational media is significantly important because learners are able to see relationships among representations (i.e., *representational fluency*) of different mathematical instantiations. Representational fluency involves transforming or transferring from one representation to another (Goldin, 2003). Zbiek, Heid, Blume, and Dick (2007) note that technology tools are important in mathematics because of their externalized representations, dynamic actions, and multiple linked representations that promote representational fluency. Virtual manipulatives: a) link different forms of representation, such as symbolic, pictorial and concrete representations (e.g., the numeral "7" along with a picture of seven triangles); and, b) link different representational models (e.g., a set model showing ¼ and a region model showing ¼). The ability to translate among a variety of representations supports students' mathematical learning (Lesh, Cramer, Doerr, Post, Zawojewski, 2003; Yerushalmy, 1991).

Operations that Transform Objects

Shaffer and Kaput (1998) proposed that, "From a virtual perspective, the mathematics is not in performing the formal manipulations, but in imaginatively framing the problem in a way that uses clear and compelling representations, often interactively, leading to efficient and convincing solutions through interpretation of external action by technological tools" (pp. 110). Virtual manipulatives, allow learners to externalize mental mathematical activities through the transformation of virtual objects, thereby allowing the learner to reflect on the externalized objects and transformations. Studies on the theory of Physically Distributed Learning have shown that when students transform objects this changes their interpretations of those objects (Martin & Schwartz, 2005). Noss and Hoyles (1996) describe these learner interactions with technology as altering how knowledge is constructed and providing insights into how the learner expresses mathematical ideas which provide clues into the processes of learning.

Connections Between Models and Abstraction

Researchers report that microworlds support students in learning to develop problems, find solutions, and then develop visual proof of their solutions by observing relations between their actions and virtual objects (Durmus & Karakirik, 2006). Research on the connections between models and abstraction has focused on the embodied nature of mathematical thinking (Lakoff & Nunez, 2000). Botzer and Yerushalmy (2008) propose that "our bodies and senses play a significant role in mathematics thinking" (p. 114). The embodied nature of mathematical thinking is based on a relationship between bodily actions and semiotic activities (i.e., "activities that combine concrete actions with the construction and elaboration of signs," Botzer & Yerushalmy, 2008, p. 112), and how those actions and activities are mediated by technological artifacts. These ideas have roots in the Vygotskian notion of semiotic mediation which "maintains that cognitive functioning is intimately linked to the use of signs and tools, and is affected by it" (Botzer & Yerushalmy, 2008, p. 114). Specifically, Martin and Schwartz's (2005) theory of Physically Distributed Learning (PDL) examines this connection between actions and manipulatives. The PDL theory suggests that "the emergence of new interpretations through physical adaptations of the environment can be an important benefit of physical action for learning abstract ideas" (Martin & Schwartz, 2005, p. 589). The applications of this to virtual manipulatives are that students' physical interactions with the virtual manipulatives provide opportunities for a process of adaption and reinterpretation of the externalized representations, thereby promoting development and learning.

RESEARCH QUESTIONS

This meta-analysis was guided by two primary research questions with sub-questions:

1. What are the effects of virtual manipulatives as an instructional treatment on student achievement in mathematics? Sub-questions examined virtual manipulatives: a) in comparison with physical manipulatives; b) by mathematical domains, grade levels, and study durations; and, c) student subgroups (gender and achievement).
2. Is there empirical evidence that specific affordances of virtual manipulatives promote student learning in mathematics?

METHODS

The study used a meta-analysis design to compute effect size scores from empirical research studies that reported student achievement results on posttests of mathematical content knowledge in each study. Effect size scores were summarized and analyzed holistically and in sub-categories. A conceptual analysis was used as a secondary method to identify researcher-reported affordances of virtual manipulatives that promoted student learning.

Data Sources

The meta-analysis began with an exhaustive search of electronic databases and a manual search of peer-reviewed publications using standard search criteria (Boote & Beile, 2005). The process included library searches (electronic and manual) in databases such as ERIC, PsycInfo, Dissertation Abstracts, Web of Science, Google Scholar, and Social Sciences Index. Search terms included: virtual manipulatives, dynamic manipulatives, computer manipulatives, virtual tools, mathematics manipulatives, mathematics tools, technology tools, computer tools, mathematics applets, and computer applets.

Criteria for Inclusion in the Meta-Analysis

In this meta-analysis, we focused only on those studies that reported results on the use of virtual manipulatives as defined by Moyer et al. (2002) (i.e., interactive, visual representations of dynamic objects). Therefore, the studies included in this meta-analysis were only those where learners had the opportunity to manipulate movable computer representations of objects for mathematics learning. General computer-based microworlds and the broader category of digital learning objects were not the focus of this study.

A number of studies were reviewed and not included in the meta-analysis because they did not meet our standards. Of 150 publications initially identified, 82 articles and dissertations were empirical studies (meaning that the researchers collected data during the study), 76 of these were peer reviewed. The other publications were opinion articles, theory papers, and instructional strategies, and these were not included in the analysis. Studies on geometry software programs (e.g., Geometer's Sketchpad, Cabri, and GeoGebra; Battista, 2007), and teachers' use of virtual manipulatives (e.g., Bouck & Flanagan, 2010; Moyer, Salkind, & Bolyard, 2008) were also excluded because they did not focus on student learning. Nine studies were excluded because they did not use virtual manipulatives for mathematics instruction. Seven additional studies were removed after coding all studies on a scale of 0 to 3 and evaluating them for seven experimental threats to internal validity: history, mortality, instrumentation, testing, selection, regression, and maturation (Gall, Gall & Borg, 2003). This left 66 studies that met our criteria. These 66 studies contained minimal threats to validity (i.e., not at a level to warrant the removal of the studies from our analysis). These 66 studies were also consistent in their instructional approach to the use of the virtual manipulatives. For example, in these studies students worked individually or in pairs at computers with the virtual manipulatives.

ANALYSIS

Effect Size Analysis

An effect size reports the magnitude of treatment effects for each comparison in a research study and is independent of sample sizes. Of the 66 potential studies that met our criteria for inclusion, 32 studies included data from which effect sizes could be calculated. In the 32 studies, we identified 82 effect cases which compared virtual manipulatives with other instructional treatments. Effect size scores were determined by: a) comparing gain scores for each effect using pre- and post-test scores to determine Cohen's d; b) using the difference between the post tests and dividing by the pooled standard deviation to determine Cohen's d; and, c) converting F values from ANOVAs. Effect sizes were summarized across the 82 cases to determine overall averaged effects and effects in sub-categories.

Conceptual Analysis

The conceptual analysis identified specific researcher-reported affordances of the virtual manipulatives that promoted student learning in mathematics. Sixty-six studies met our criteria for inclusion in the conceptual analysis. Studies were read in their entirety. Readers identified empirical results reported by researchers as explanations for improved student learning when using virtual manipulatives. Readers used data reduction to organize the individual results from each study, using a matrix to form conceptual categories of affordances (Miles & Huberman, 1994). Our interpretation of the data was guided by hypothesized affordances posited by Sarama and Clements (2009).

RESULTS

The first section of the results reports averaged effect size scores for virtual manipulatives as an instructional treatment on student achievement, focusing on overall effects and effects in sub-categories. The second section reports on the empirical evidence that five specific interrelated affordances of virtual manipulatives promote student learning in mathematics, based on the conceptual analysis.

Averaged Effect Size Results: Virtual Manipulatives vs. Other Instructional Treatments

The effect sizes for each comparison are reported as an average of the effects yielded from each case. An effect size of less than 0.20 is considered to be small, effect sizes in the range of .25 to .75 are considered to be moderate, and those over .80 are considered large (Urdan, 2010). The 82 effect size scores and comparisons are presented in Table 1. The comparison for virtual manipulatives (used alone or in combination) vs. all other instructional treatments yielded a moderate effect (0.35). The comparison for virtual manipulatives (only) vs. other instructional treatments produced a moderate effect (0.34), virtual manipulatives (only) vs. physical manipulatives produced a small effect (0.15), and virtual manipulatives (only) vs. classroom instruction using textbooks produced a moderate effect (0.75). The comparison for virtual and physical manipulatives (VM/PM combined) vs. other instructional treatments yielded a moderate effect (0.33), VM/PM combined vs. virtual manipulatives (only) yielded a moderate effect (0.26), VM/PM combined vs. physical manipulatives (only) yielded a small effect (0.20), and VM/PM combined vs. classroom instruction using textbooks yielded a moderate effect (0.69). Overall, the largest effects were reflected for virtual manipulatives (only) vs. classroom instruction using textbooks, and for VM/PM combined vs. classroom instruction using textbooks.

Mathematical Domains

The next analysis examined 73 effect size scores in various mathematical domains (see Table 2). The domains Fractions, Numbers and Operations, Geometry, and Measurement all yielded moderate effects (0.53, 0.42, 0.41, and 0.40, respectively) for virtual manipulatives vs. other instructional treatments. The domain with the largest number of effect size score cases was Geometry (26). Overall, the largest effect was in the domain of Fractions. The "other" category contained six effect size scores that focused on problem solving, logic, and probability.

Grade Level Groups

The next analysis was conducted by grade level groups, using two grades per group, with the exception of Grades 9 through 12 (see Table 3). When studies spanned more than one group, effect sizes were entered into each group. The majority of studies using virtual manipulatives were conducted in Pre-Kindergarten through Grade 6, with relatively few studies at the middle school, high school, and university levels. Virtual manipulatives vs. other instructional treatments at the Pre-Kindergarten, 1-2, and 3-4 levels yielded a moderate effect (0.34, 0.56, and 0.37 respectively); while studies in Grades 5-6 yielded a small effect (0.11); Grades 7-8 yielded a negative effect (-0.02); and Grades 9-12 and the University level yielded large effects (0.76 and 1.17, respectively). It is important to note that two of the three effect sizes obtained for Grades 9-12 were from studies investigating unique groups and modules (e.g., special education students and individualized on-line algebra modules).

Duration of Treatment

Table 4 aggregates the effects into "length of treatment" based on the duration that students participated in virtual manipulative treatments. Almost half of the studies were conducted for more than 10 days. The shortest length of treatment (1 day) yielded essentially no effect (0.003)

Table 1. Effect size scores for virtual manipulatives compared with other treatments

Comparisons	*N* Comparisons	Effect Size
Virtual Manipulatives (alone or in combination)		
vs. Other Instructional Treatments	73	0.35
Virtual Manipulatives (only)		
vs. Other Instructional Treatments	56	0.34
vs. Physical Manipulatives (only)	38	0.15
vs. Classroom Instruction (with textbooks)	18	0.75
Virtual and Physical Manipulatives (combined)		
vs. Other Instructional Treatments	26	0.33
vs. Virtual Manipulatives (only)	9	0.26
vs. Physical Manipulatives (only)	11	0.20
vs. Classroom Instruction (with textbooks)	6	0.69

Table 2. Effect size scores for virtual manipulatives by mathematical domains

Mathematical Domains	*N* Comparisons	Effect Size
Fractions	11	0.53
Numbers and Operations	10	0.42
Geometry	26	0.41
Measurement	6	0.40
Integers	4	0.26
Algebra	10	0.16
Other	6	-0.04

for virtual manipulatives vs. other instructional treatments. Treatments of 3-5 days yielded a small effect (0.21), while treatments of 2, 6-10, and more than 10 days yielded moderate effects (0.36, 0.46 and 0.49 respectively).

Limited Results on Virtual Manipulatives with Student Sub-Groups

Only 7 of the 66 studies examined the use of virtual manipulatives by students of differing academic abilities. For example, two studies found significant pre to post test gains, but no significant differences among low, medium and high ability groups (Drickey, 2000; Kim, 1993). In contrast, three other studies found that gains were greater for the low achieving groups using virtual manipulatives (Hativa & Cohen, 1995; Lin, Shao, Wong, Li, & Niramitranon, 2011; Moyer-Packenham & Suh, 2012). Only 5 of the 66 studies investigated gender differences when virtual manipulatives were used, and these produced inconclusive results. Only 3 of the 66 studies focused on special needs students. With so few studies reporting effects on different ability groups, gender, and special needs students, there is not sufficient evidence to draw any conclusions about student subgroups. Further research on student sub-groups should seek to understand how different technology interfaces and affordances provide access to the mathematics for different students.

What Specific Affordances of Virtual Manipulatives Promote Student Learning?

The results of the conceptual analysis identified researcher-reported empirical evidence for

Table 3. Effect size scores for virtual manipulatives by grade level groupings

Grades	*N* Comparisons	Effect Size
Pre-Kindergarten	11	0.34
1-2	17	0.56
3-4	29	0.37
5-6	22	0.11
7-8	5	-0.02
9-12	3	0.76
University	2	1.17

Note: When the study spanned more than one group, the effect size was entered into each group.

Table 4. Effect size scores for virtual manipulatives by length of treatment

Length of Treatment	*N* Comparisons	Effect Sizes
1 day	12	0.00
2 days	5	0.36
3-5 days	12	0.21
6-10 days	12	0.46
More than 10 days	31	0.49

five specific interrelated affordances of virtual manipulatives that promote student learning in mathematics: *focused constraint*, *creative variation*, *simultaneous linking*, *efficient precision*, and *motivation*. These categories have some commonalities with the "seven hypothesized, interrelated affordances" proposed by Sarama and Clements (2009, p. 147), providing additional evidence for their empirical validity. An important thread that ties these categories together is how the representations, the learner and the mathematics interact virtually.

Virtual Manipulatives Focus and Constrain Student Attention on Mathematical Objects and Processes

(17 studies). One affordance identified during the conceptual analysis was *focused constraint.* Constraining and focusing features included: bringing to a specific level of awareness mathematical aspects of the objects which may not have been observed by the student; and, applets focusing student attention on specific characteristics of mathematical processes or procedures. A specific example of this affordance in action is reported by Evans and Wilkins (2011) in their extensive observations and interviews over a two-week period with 12 second graders engaged in geometry problem solving tasks. This study reported that, because of the technology interface, children were required to spend time exploring the interface (i.e., how to move, rotate, grasp, and flip the geometric shapes) before they could engage in the problem solving tasks. This process afforded "participants more focused reflection on geometric transformations as a result of their deliberate alignment of the conceptual action to the interface…" (Evans & Wilkins, 2011, p. 164). Evans and Wilkins (2011) noted that children in their study using physical manipulatives focused on the problem solving task, while in contrast, children using the virtual manipulatives focused on the underlying geometric concepts to solve the problem because of the initial constraints of the technology interface. Researchers concluded that this finding was the result of virtual manipulatives constraining students' physical interactions with the manipulatives and, therefore, more communication and explicit discussion was necessary when students worked in triads.

The focused constraint affordance supports the processes of computation and abstraction. For example, Manches, O'Malley, and Benford (2010) reported that the constraints in the virtual condition in their study (compared with the physical condition) influenced the types of strategies children used with more frequency (commutative versus compensation). In other words, because the students in the virtual condition were constrained to moving one object at a time, they exhibited more compensation strategies in the virtual manipulatives condition (i.e., adding or subtracting one block at a time). One benefit of constraining the learner's actions through technology is that it makes mathematical properties and relationships explicit for learners (Durmus & Karakirik, 2006; Zbiek et al., 2007). Sarama and Clements' (2009) describe this as "bringing mathematical ideas and processes to conscious awareness" (p. 147). The dynamic visual nature of the representations focuses students on engaging with virtual objects and constrains their attention to mathematical processes presented through the representations.

Virtual Manipulatives Encourage Creativity and Increase the Variety of Students' Solutions

(14 studies). The second affordance identified during the conceptual analysis was *creative variation*. This affordance allows students to generate their own representations, encourages creativity and novelty, and prompts experimentation. For example, Trespalacios's (2010) third graders used virtual manipulatives to generate their own representations, resulting in significant gains between the pre-test and post-test, and maintaining these scores on a delayed post-test. In other studies, students used more elements in their patterns and exhibited more creative behaviors when using virtual blocks compared with physical blocks (Lane, 2010; Moyer, Niezgoda & Stanley, 2005); and, preschool students experimented to develop representations, and this experimentation yielded more patterns and more transformations in their patterns (Clements et al., 2001). Sarama and Clements' (2009) describe this type of creative behavior as, "changing the very nature of the manipulative" (p. 148). In this category the virtual manipulatives support students' development of their own representations. Noss and Hoyles (1996) provide powerful evidence that putting students in situations where they create their own models and mathematical generalizations is a pedagogical approach that supports mathematical construction and exploration.

These results support some of the earliest research by Thompson (1985) in a course for elementary school teachers who had been using the *Motions* microworld for two semesters. Thompson found that classes that used the *Motions* microworld showed a much greater tendency to play with problems and explore the technology before rushing to the solution of the problem. The researcher reported that students in a computer microworld are more likely to experiment with a technology tool to see what will happen than when they are solving problems using paper and pencil methods.

Virtual Manipulatives Simultaneously Link Representations with Each Other and with Students' Actions

(20 studies). The third affordance identified during the analysis was *simultaneous linking*. This category of affordances included research studies that reported: linking two different dynamic pictorial objects, linking dynamic pictorial objects with symbols, and linking the student's movement of the computer mouse with the motion of dynamic pictorial objects. Virtual manipulatives link the learner with the representations, allowing the learner to experience and interact with multiple embodiments of concepts (i.e., graphical, abstract, and dynamic) (Lakoff & Nunez, 2000; Botzer & Yerushalmy, 2008). Botzer and Yerushalmy (2008) describe the embodied nature of mathematical thinking as "activities that combine concrete actions with the construction and elaboration of signs" (p. 112). Sarama and Clements describe this as "linking the concrete and the symbolic with feedback" and "supporting mental actions on objects" (2009, p. 147-48). Suh and Moyer (2007) reported students observing the links between algebra symbols and the movement of a balance scale. Similarly, Beck and Huse (2007) reported that simultaneous and linked representations focused students to observe that as the spinner changed the graph changed.

Virtual manipulatives enable embodied concrete action to be integrated with semiotic activity because they are grounded in physical action (i.e., the use of the computer mouse or the touch pad), and because they include physical motion linked with mathematical representations (Martin & Schwartz, 2005). Linking two or more representations facilitates students' connections among different representations of the same concept (e.g., a picture of a region with one-fourth shaded and the numeric representation "¼"), thereby increasing students' representational fluency (Zbiek et al., 2007). Students preferred the linked pictorial/symbolic applets because problems were written for them on the screen, they did not have to recount the number of blocks, and they could see the result of their actions (i.e., the numbers changed) (Haistings, 2009). Researchers noted higher scores on assessments when students were more actively involved in manipulating the virtual manipulatives (Bodemer, Ploetzner, Feuerlein, & Spada, 2004). When linked representations change simultaneously, responding to students' actions with visual feedback, students have observable evidence of their actions, and this provides learners with opportunities to adapt and reinterpret the representations (Martin & Schwartz, 2005). In some cases the representations are reflecting the thinking of the learner, while in other cases the representations are reflective of the ways in which learners think about the mathematics (i.e., cognitive fidelity; Zbiek et al., 2007).

Virtual Manipulatives Contain Precise Representations Allowing Accurate and Efficient Use

(25 studies). The fourth affordance identified during the analysis was *efficient precision*. Virtual manipulatives are faithful to mathematical properties, provide precise mathematical examples, and create multiple copies of dynamic objects efficiently, therefore exhibiting mathematical fidelity (Zbiek et al., 2007). Sarama and Clements describe this as "encouraging and facilitating complete, precise explanations" (2009, p. 147). In this category, the representations exhibit accuracy and efficiency in their presentation allowing students to learn concepts at a significantly faster rate. Several contributing factors were reported: the ease of manipulation which negated problems of fine motor skills and permitted students to focus on concepts (Terry, 1995); efficiency increases practice time, allowing students to be more engaged with the mathematics (Beck & Huse, 2007; Yuan, Lee, & Wang, 2010); ability to change and rearrange the virtual blocks, which minimized manipulation time (Izydorczak, 2003); and speed of applet features, which produced quicker solutions (Beck & Huse, 2007). Students using virtual manipulatives were more methodical and purposeful than students using physical manipulatives, leading to more mathematically accurate answers.

This affordance was described by Marrades and Gutierrez (2000) about dynamic geometry manipulatives that generate numerous examples easily so that students can perform transformations on those examples, an action not as easily and efficiently matched in a static environment. Accuracy is also important in Schoenfeld's (1983) axiom of his students' geometry learning which states, "Insight comes from very accurate drawings. The more accurate the drawing, the more likely one is to derive useful information from it" (p. 338). The virtual manipulatives made it possible for students to perform repetitive tasks quickly and, therefore, spend more time interacting with the objects and the mathematics. Accuracy of presentation and efficiency of manipulation enabled students to quickly develop and organize multiple accurate examples, making it easier for students to identify mathematical patterns.

Virtual Manipulatives Motivate Students to Persist at Mathematical Tasks

(30 studies). The fifth affordance identified during the analysis was *motivation*. This feature of the virtual manipulatives impacts student learning through students' affective responses (i.e., VMs were enjoyable), student interest (i.e., VMs maintain students' attention), and student engagement (i.e., students persist longer at mathematical tasks). Clements et al. (2001) reported that students enjoyed working with the computers so much that they drew a chalk replica of the applet on their playground. Interviews revealed that students believed the virtual manipulatives were easier and faster to use than paper and pencil (Reimer & Moyer, 2005), and virtual manipulatives were more "game like" while physical manipulatives were

more like work (Hsiao, 2001). Lane (2010) reported that third grade students using virtual manipulatives were on task 20% more of the time than those using physical manipulatives.

Although motivation was one of the most commonly identified features reported to impact student learning when using the virtual manipulatives, motivation itself seems to be under-emphasized as an affordance of the technology because of its seeming link with affect rather than cognition. Yet, several researchers argue that there is a much more intertwined relationship among affect, motivation, and cognition. For example Goldin views affect "as an essential structural part of learning mathematics – rather than as peripheral to cognitive aspects of learning" (as cited in Lesh & Zawojewski, 2007, p. 777). In the report of the National Mathematics Advisory Panel (2008), the Task Group on Learning Processes emphasized the important role that motivation plays in learning based on years of established research. Motivation relates to learner engagement and self-efficacy impacting student achievement outcomes. One possible explanation for the motivational appeal of virtual manipulatives may be found in the theory of psychological essentialism. This theory suggests that things (e.g., virtual manipulatives) have an underlying reality (or *essence*) that cannot be observed directly, and when an essence is unknown to the student, students are naturally motivated to make sense of the essence of the object (Gelman, 2004).

DISCUSSION

The purpose of this comprehensive meta-analysis was to synthesize results on the effects of virtual manipulatives on student achievement. The results indicate that virtual manipulatives yield a moderate effect when compared with other instructional treatments. There were also positive effects when virtual manipulatives were examined in comparison with other instructional treatments by mathematical domain, grade level, and study duration. Results of the conceptual analysis identified researcher-reported empirical evidence for five specific affordances of virtual manipulatives that promote student learning in mathematics. These results are discussed in the sections that follow.

What is the Effect of Virtual Manipulatives on Student Achievement in Mathematics?

Virtual manipulatives have a moderate effect on student achievement when compared with other instructional treatments. These moderate effects are constant when virtual manipulatives are used alone or in combination with physical manipulatives. Of note are two important results. The larger effects among these results were yielded for virtual manipulatives alone and for VM/PM combined when these were compared with classroom instruction using textbooks. Yet, when virtual manipulatives alone or VM/PM combined were compared with physical manipulatives alone, this yielded small effects. These results suggest that the virtual manipulatives, as well as VM/PM combined, have unique embodiments that have positive impacts on student achievement in mathematics. As Dienes' (1973) theory of embodied knowledge proposed, learners understand abstract concepts by interacting with multiple embodiments of those concepts. Critical in the use of multiple embodiments is that the learner begins to recognize generalized concepts and abstractions across the embodiments, rather than focusing on the irrelevant features in each specific embodiment (Lakoff & Nunez, 2000). A long history of mathematics education research has concluded that when learners attempt to make connections among different representational embodiments (i.e., different virtual representations of the same concept or virtual and physical manipulatives as representations of the same concept) this supports concept development and mathematical abstraction (Confrey & Smith, 1994; Lesh et al., 2003; Yerushalmy, 1991).

What are the Effects of Virtual Manipulatives by Domain, Grade, and Duration?

The results revealed moderate effects for virtual manipulatives compared with other instructional treatments in a variety of mathematical domains including fractions, geometry, numbers and operations, measurement, and integers. These results suggest that the virtual manipulatives allowed students to make mathematical connections across a variety of abstract and graphical representations within multiple mathematical domains. Of note is that about one-third of the studies comparing virtual manipulatives are in the domain of geometry.

The analysis by grade levels indicated mixed effects across the grade levels groups, with moderate effects for Preschool through Grade 4, small effects for Grades 5-6, no effect for Grades 7-8, and large effects for Grades 9-12 and University-level. However, the large effects for Grades 9-12 and University-level students were based on only five effect size scores. Further research on uses of virtual manipulatives with students beyond Grade 6 would be an important contribution to the virtual manipulatives research agenda.

The analysis of treatment durations yielded a generally increasing trend in the averaged effect size scores. Virtual manipulative treatments of 1 day and 3-5 days yielded small effects, while 2 days, 6-10 days, and more than 10 days yielded moderate effects. Almost half of the effect sizes came from studies where students used virtual manipulatives for more than 10 days. The use of virtual manipulatives for longer treatment durations allows the novelty of working with computers to decline and allows students time to learn to work with the virtual manipulatives (e.g., how to access the websites, how to manipulate the objects in the microworld, how to save and print work). Studies that examine the effects of virtual manipulative use over a semester, a year, or multiple years would be important contributions to the research.

What Technology Affordances Promote Learning?

The conceptual analysis revealed evidence of five interrelated affordances of virtual manipulatives that promote student learning: *focused constraint*, *creative variation*, *simultaneous linking*, *efficient precision*, and *motivation*. In the *focused constraint* affordance, representations are used to support the processes of computation and abstraction; while in *creative variation*, virtual manipulatives support the development of students' own representations. In the affordances *simultaneous linking*, *efficient precision*, and *motivation*, representations act as linking mechanisms, exhibit accuracy and efficiency, and capture students' interest, encouraging persistence. Understanding the influence of the ways in which representations are used in virtual manipulatives, and the ways in which learners interact with the representations, may provide direction on the use of virtual manipulatives for mathematics teaching. The field would benefit from further research on how virtual manipulatives focus, constrain, direct, guide, and highlight mathematical and pictorial aspects of representations. Given the results on affordances as both constraining and open-ended, future research may pursue understanding the balance between these seemingly dichotomous features.

DIRECTIONS FOR FUTURE RESEARCH

What new inquiries are needed for the research agenda on virtual manipulatives to move forward? As suggested previously, there are areas of research yet to be explored including: research on virtual manipulatives with students beyond Grade 6, research that seeks to understand how different affordances provide access to the mathematics for different students, research on the balance between virtual manipulatives that focus and constrain learners while allowing for creativity and exploration. Because applets vary in the amount of constraint and support they offer to learners, research is needed to determine which applet characteristics are most effective for teaching specific concepts and for whom. For example, Bolyard (2006) compared the effectiveness of two applets in teaching the addition and subtraction of integers and Haisting (2009) compared the effectiveness of applets with and without symbolic linking.

These types of studies, specifically focusing on elements within the applets, have the potential to contribute new insights on those affordances that effect student achievement for different students under different conditions.

One research area that was beyond the scope of this paper is the research on effective instructional practices for using virtual manipulatives (Moyer-Packenham, 2010). For example, which applets can be used to effectively introduce concepts, explore concepts, or practice concepts? What teacher actions or instructional materials enhance student use of virtual manipulatives? Many questions about teaching with virtual manipulatives are yet to be explored.

CONCLUSION

Virtual manipulatives allow students to explore mathematical ideas in ways that are very different than typical paper and pencil activities, providing opportunities for different kinds of mathematical observations and learning to occur. As the cumulative research to date indicates, these different mathematical experiences produce moderate effects on student achievement when compared with other instructional treatments. The five interrelated affordance categories of virtual manipulatives help us to understand, in part, why virtual manipulatives impact student learning for different students in different ways. An important thread that ties the affordance categories together is how the representations, the learner and the mathematics interact with the virtual manipulatives.

To advance the study of any particular area of research, it is important to synthesize the current evidence in that area and provide direction for further inquiries. This study serves as a foundation on which to base future research questions on virtual manipulatives by identifying what has been accomplished and what is yet to be understood. This meta-analysis also emphasizes the need for communication and collaboration among researchers and practitioners across the mathematics and technology communities. The design and implementation of virtual manipulatives for mathematics instruction requires a close examination of specific features of the tools and the influence of those features on individual students. We hope that this study provides our colleagues in the mathematics and technology communities with a common foundation on which to form the next decade's questions on the use of virtual manipulatives for mathematics teaching and learning.

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